Quiz 10 Solution

March 7, 2018

1. (2 points) The critical values of $f(x)$ are $x = 0$ and $x = 4$. If the second derivative is $f''(x) = \sec(\frac{x\pi}{3})$, what can we say about the relative extrema of $f(x)$?

   (a) There is a rel. max. at both $x = 0$ and at $x = 4$

   (b) There is a rel. min. at both $x = 0$ and $x = 4$

   (c) There is a rel. min. at $x = 0$ and a rel. max. at $x = 4$

   (d) There is a rel. max. at $x = 0$ and a rel. min. at $x = 4$

   (e) There is not enough information.

   Solution: Since we only know the second derivative, we'll use second derivative test. Since $f''(0) = \sec(0) = 1 > 0$, $f(x)$ is concave up near $x = 0$, and therefore there is a minimum at $x = 0$.
   Since $f''(4) = \sec(\frac{4\pi}{3}) = -2 < 0$, $f(x)$ is concave down near $x = 4$, and therefore there is a maximum at $x = 4$.

   Answer: (c)

2. (2 points) Find the absolute extrema of $f(x) = \dfrac{x^3}{3} - 6x^2$ on the interval $[-1, 3]$.

   Solution: First, we find the critical values in the interval $[-1, 3]$:

$$\begin{aligned} f'(x) = x^2 - 12x &\overset{\text{set}}{=} 0 \\ x(x-12) &= 0 \\ x = 0,\ \cancel{x = 12}\ &\text{(since 12 is not in the interval } [-1,3]) \end{aligned}$$

   Now, we find the values of $f(x)$ at the critical value $x = 0$ and at the endpoints:

x	-1	0	3
$f(x)$	-19/3	0	-45

   So the absolute minimum is at $(3, -45)$ and the absolute maximum is $(0, 0)$.

   Answer: Absolute min: $(3, -45)$; absolute max: $(0, 0)$.

3. (1 point) What are your plans for break?

   Answer: Answers will vary.